AN OUTLINE OF PLANE GEOMETRY AS USED IN THE DURFEE HIGH SCHOOL.

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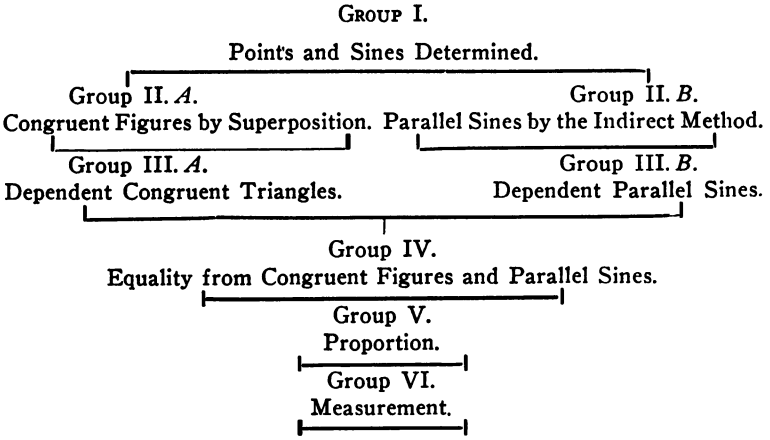

In group I. are:

(*a*) Through a point only one straight line can be drawn making a given angle with a second straight line.

(*b*) Through two points only one straight line can be drawn.

(*c*) Through a point only one straight line can be drawn perpendicular to a second straight line.

(*d*) Through a point only one straight line can be drawn parallel to a second straight line.

(*e*) Two straight lines can intersect in only one point.

(*f*) Only one circle can be drawn with a given center and a given radius.

a needs a word of explanation. If one line crosses a second line, four angles are formed but *the angle* between the lines is that angle formed by the revolution of the first line, counter-clockwise, from the position of the second line. There are two cases in *a*:

1. Point on the line; 2. Point outside the line.

(b) is a special case of (a), 1 with a zero angle.
(c) is a special case of (a), 1 and 2.
(d) is a special case of (a), 2 with a zero angle.

Group I. is usually assumed and sometimes the two basic theorems of measurement, the measure of a central angle, and of a rectangle. In a few courses group II. A and B are also assumed.

In group II. A are:

(a) All straight angles are equal.

(b) If two triangles have two sides and the included angle of the one equal respectively to two sides and the included angle of the other, the triangles are congruent.

(c) Similar to (b), but with two angles and a side.

(d) If two parallelograms have two adjacent sides and the included equal, etc.

(e) A diameter of a circle bisects the circle.

(f) In the same or equal circles, equal central angles intercept equal arcs.

(g) Converse of f.

These theorems are proved by superposition and group I.

(c) includes "two angles and the included side" and can be proved by (a) in group I.

(f) may be worded: Two sectors are congruent if they have two straight sides and the included angle equal, etc.

(g) may be worded: Two sectors are congruent if they have three sides equal, etc.

In group II. B are:

(a) If two straight lines are cut by a transversal making the corresponding angles equal, the lines are parallel.

(b) Two straight lines parallel to a third straight line are parallel.

These are proved by the indirect method and group I.

(a) can be proved by (a) in group I.; (b) by (d).

In group III. the dependent cases of congruent triangles are the right triangles and the scalene triangles having three sides of the one equal to three sides of the other. They all depend upon (b) or (c) in group II. A.

The common theorems of parallel lines depend directly or indirectly upon (*a*) in group II. *B*.

Group IV. is equality derived from congruent figures or from parallel lines. The first theorem of equal angles from parallel lines can be either alternate-interior angles or corresponding angles. The proof is indirect, using (*d*) in group I.

In group V. is proved the last important theorem for parallel lines: If a line divides two sides of a triangle proportionally, it is parallel to the third side. The proof is indirect.

The four cases using the indirect proof, mentioned above, involve parallel lines and use the simplest elementary principles as group I. or the general axioms. In no other theorems is the indirect method favored except, perhaps, in one or two cases of inequality and here, even, it can usually be avoided.

A, 1 in group I. is not true in the geometry of space and is the basis of the plane theorems that are not necessarily true in space. The common theorems of this kind are:

(*a*) Group I., *c*.

(*b*) Two perpendiculars to the same straight line are parallel.

(*c*) A straight line perpendicular to a radius at its end is tangent to the circle.

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